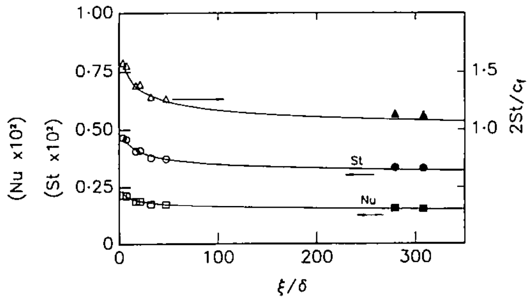

FIG. 4. Stanton number St, Nusselt number Nu and Reynolds analogy factor $2St/c_f$. Closed symbols refer to a thermally fully developed flow (I). ○, experimental values of St; △, $2St/c_f$; □, Nu; ——, equation (1).

however that this difference would have a significant effect on (1), which has been validated in the boundary layer when $\Delta_H \ll \Delta$ [9]. Assuming that the present value of Pr ($\simeq 0.72$) is sufficiently close to $Pr = 1$, the close agreement between (1) and the present values of St (Fig. 4) indirectly suggests that the assumption $Pr_T = 1$ (or Reynolds' analogy) should be reasonable, at least in the near-wall region, for the present situation. Direct numerical simulation data in the near-wall region of a thermally fully developed channel flow [10] have confirmed the validity of $Pr_T = 1$ (when Pr is near 1). It would certainly be of interest to extend this conformation to a developing thermal layer.

CONCLUSIONS

A step change in heat flux has been applied to one of the walls of a fully developed turbulent channel flow, while the other (opposite) wall is at approximately ambient temperature. In the near-wall region, scaling on wall variables is satisfied to a good approximation by the mean temperature but not by the r.m.s. temperature. Sufficiently downstream of the step, mean and r.m.s. temperature distributions asymptote to values obtained for a thermally fully developed flow. The streamwise variation of the Stanton number, Reynolds analogy factor and Nusselt number downstream of the heating origin is well described by an empirical relation obtained for a boundary layer with an unheated starting length.

REFERENCES

1. W. C. Reynolds, W. M. Kays and S. J. Kline, Heat transfer in the turbulent incompressible boundary layer: parts I, II and III, NASA Memos 12-1-58W, 12-2-58W and 12-3-58W (1958).
2. R. A. Antonia, H. Q. Danh and A. Prabhu, Response of a turbulent boundary layer to a step change in surface heat flux, *J. Fluid Mech.* **80,** 153–177 (1977).
3. L. R. Saetran, Turbulent boundary layer with a step in the wall temperature. In *Forum on Turbulent Flows—1989* (Edited by W. W. Bower and M. J. Morris), FED Vol. 76, pp. 107–114. American Society of Mechanical Engineers (1989).
4. R. P. Taylor, P. H. Love, H. W. Coleman and M. H. Hosni, Heat transfer measurements in incompressible turbulent flat plate boundary layer with step wall temperature boundary conditions, *J. Heat Transfer* **112,** 245–246 (1990).
5. L. W. B. Browne and R. A. Antonia, Calculation of a turbulent boundary layer downstream of a sudden decrease in surface heat flux or wall temperature, *Proc. Third Symp. on Turbulent Shear Flows*, University of California at Davis, pp. 10.18–10.22 (1981).
6. M. Teitel and R. A. Antonia, Heat transfer in a fully developed turbulent channel flow: comparison between experiment and direct numerical simulations, *Int. J. Heat Mass Transfer* **36,** 1701–1706 (1993).
7. M. Teitel and R. A. Antonia, The interaction region in a turbulent duct flow, *Physics Fluids A* **2,** 808–813 (1990).
8. J. C. Lecordier, A. Dupont, P. Gajan and P. Paranthoen, Correction of temperature fluctuation measurements using cold wires, *J. Phys. E: Sci. Instrum.* **17,** 307–311 (1984).
9. W. M. Kays, *Convective Heat and Mass Transfer.* McGraw-Hill, New York (1966).
10. R. A. Antonia and J. Kim, Turbulent Prandtl number in the near-wall region of a turbulent channel flow, *Int. J. Heat Mass Transfer* **34,** 1905–1908 (1991).

Int. J. Heat Mass Transfer. Vol. 36, No. 6, pp. 1709–1713, 1993
Printed in Great Britain

0017-9310/93 $6.00 + 0.00
Pergamon Press Ltd

Transfer function method for analysis of temperature field

J. GOŁĘBIOWSKI,† A. SKOREK‡ and A. J. JORDAN†

† Technical University of Białystok, Poland
‡ Université du Québec à Trois-Rivières, Canada

INTRODUCTION

IN RECENT years a number of articles have been written on the wave model of heat propagation where the finite velocity of the heat wave is taken into account [1–7]. The wave model of heat propagation leads to a precise analysis of many physical phenomena which, when analysed by Fourier's law, will result in some errors [2].

Classical examples of the correctness of the heat wave model are intensive heating of solids by means of laser wave impulses of high amplitude and short duration [8], electromagnetic radiation [9], fast heat flow in rarefied media [10], etc.

Fourier's law, when used in the classical analysis of thermal problems, defines the dependence between heat flux intensity and time–space distribution of temperature T. By combining Fourier's law with the principle of energy conservation we obtain a parabolic equation of heat diffusion

$$\alpha \nabla^2 T = \frac{\partial T}{\partial t} \tag{1}$$

where $\alpha = k/\rho c$ is the diffusion coefficient and k, ρ and c are thermal conductivity, mass density and specific heat, respectively. A physical interpretation of the solution of equation (1) shows that the speed of heat propagation is infinite. In some of the cases mentioned above, there is, by necessity, a generalization of the mathematical model represented by equation (1). To achieve this we use a model of heat wave damping. Then Fourier's law undergoes modification and, in combination with the principle of energy conservation, results in a heat propagation model in the form of a hyperbolic equation [1, 2, 8]

$$\alpha\nabla^2 T = \frac{\partial T}{\partial t} + \tau\frac{\partial^2 T}{\partial t^2} \tag{2}$$

where τ is the time of thermal relaxation. The solution of equation (2) can be interpreted as a superposition of damped heat waves propagating at a finite speed $a = \sqrt{(\alpha/\tau)}$.

Recent papers [1–10] have shown differences in heat propagation resulting from the analysis of the parabolic model and hyperbolic model. The analysis was conducted only in the time domain. A question arises here in what conditions we can use the wave model of heat propagation, and which conditions are appropriate for the diffusion model. This article makes an attempt to answer this question.

Taking thermal problems occurring in semiconductors [11] into consideration, we shall assume a semi-infinite medium whose thermal field is excited by an external heat flux. Having defined the transfer function of the analysed system the analysis will be carried out in the frequency domain.

DYNAMIC PROBLEMS OF HEAT FLOW

We shall consider here a half-space representing a semi-infinite solid. In this half-space the temperature field $T(x, t)$ is excited by an external heat flux $q_0(t)$.

We shall calculate the transfer function of a system defined in the following way

$$K(x,s) = \frac{\bar{T}(x,s)}{Q_0(s)} \tag{3}$$

where $Q_0(s) \Leftrightarrow q_0(t)$ is a Laplace transform of the surface heat flux $q_0(t)$ (input signal) and $\bar{T}(x,s) \Leftrightarrow T(x,t)$ is a Laplace transform of a temperature $T(x,t)$ at an x coordinate point (output signal).

The determination of the transfer function (equation (3)) of the system considered is convenient for the following reasons:

(1) the dynamic properties of the half-space will be defined independently of the form of the heat flux striking the semi-infinite medium;

(2) the values of the transfer function on the imaginary axis define both the amplitude characteristics

$$A(x,\omega) = |K(x, j\omega)|, \tag{4a}$$

and phase characteristics of the semi-infinite solid

$$\phi(x,\omega) = \arg K(x, j\omega); \quad \text{and} \tag{4b}$$

(3) Borel's theorem on the convolution of two functions will make it possible to determine the response of the half-space on the input of a given heat flux form $q_0(t)$

$$T(x,t) = \int_0^t k(x,\eta)q_0(t-\eta)\,\mathrm{d}\eta \tag{5}$$

where $K(x, s) \Leftrightarrow k(x, t)$.

Using a Laplace transform for the parabolic (1) and hyperbolic (2) models, respectively, it is possible to determine the transfer function of the analysed system.

Next, by examining the amplitude characteristics in the frequency domain connected with the parabolic and hyperbolic equations, respectively, we can determine the limit frequency above which the wave model of heat propagation is valid.

DYNAMICS OF WAVE HEAT FLOW

In half-space equation (2) is reduced to the following form

$$\alpha\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial T(x,t)}{\partial t} + \tau\frac{\partial^2 T(x,t)}{\partial t^2}. \tag{6a}$$

Moreover

$$q(x,t) = -k\frac{\partial T(x,t)}{\partial x} - \tau\frac{\partial q(x,t)}{\partial t}. \tag{6b}$$

The thermal field in the semi-infinite medium is excited by the heat flux

$$q(0,t) = q_0(t), \quad t \geqslant 0. \tag{7}$$

According to the physical aspect of the problem the role of the second boundary condition is assumed by the following limit

$$\lim_{x\to\infty} T(x,t) = 0, \quad t \geqslant 0. \tag{8}$$

Following the well-known definition of the transfer function it is necessary to introduce zero initial conditions concerning the heat flux, temperature and rate of temperature variation

$$q(x,0) = 0 \tag{9a}$$

$$T(x,0) = 0 \tag{9b}$$

$$\left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0} = 0, \quad x \geqslant 0. \tag{9c}$$

Having applied appropriate transformations and theorems (see Appendix 1) to equations (6a) and (6b) with conditions (7), (8) and (9), we obtain according to definition (3) the following form of the transfer function of heat wave motion in the semi-infinite solid

$$K_w(x,s) = \frac{\sqrt{(\alpha\tau)}}{k}\,\frac{\sqrt{\left(s+\frac{1}{\tau}\right)}}{\sqrt{s}}\exp\left(-\frac{\sqrt{\left(s\left(s+\frac{1}{\tau}\right)\right)}}{a}x\right). \tag{10}$$

On the basis of equation (10) the amplitude characteristics of the system were determined

$$A_w(x,\omega) = \frac{\sqrt{(\alpha\tau)}}{k}\sqrt[4]{\left(1+\frac{1}{\omega^2\tau^2}\right)}\exp\left[-\frac{x}{a}f(\omega)\right] \tag{11a}$$

where

$$f(\omega) = \omega\sqrt[4]{\left(1+\frac{1}{\omega^2\tau^2}\right)}\cos\left(\frac{\pi}{4}+\frac{1}{2}\arctan\omega\tau\right). \tag{11b}$$

Function (11a) is monotonically decreasing and achieves the following extreme values

$$\lim_{\omega\to 0} A_w(x,\omega) = \infty \tag{12a}$$

$$\lim_{\omega\to\infty} A_w(x,\omega) = \frac{\sqrt{(\alpha\tau)}}{k}\exp\left(-\frac{x}{2a\tau}\right). \tag{12b}$$

Phase characteristics of the system are as follows:

$$\phi_w(x,\omega) = -\tfrac{1}{2}\arctan\frac{1}{\omega\tau} - \frac{x}{a}g(\omega) \tag{13}$$

where

$$g(\omega) = \omega\sqrt[4]{\left(1+\frac{1}{\omega^2\tau^2}\right)}\sin\left(\frac{\pi}{4}+\tfrac{1}{2}\arctan\omega\tau\right). \tag{14}$$

DYNAMICS OF DIFFUSION HEAT FLOW

Diffusion heat flow is a limit case of wave heat motion, when the time of thermal relaxation tends to zero ($\tau \to 0$). Then the velocity of wave propagation $a = \sqrt{(\alpha/\tau)}$ becomes infinitely great and in equation (2), the element responsible for the wave characteristics of the phenomenon disappears. For such a case the classical model of heat transfer holds true. It assumes the form of equation (1), which for a semi-infinite medium, takes the following form

$$\alpha\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial T(x,t)}{\partial t}. \tag{15a}$$

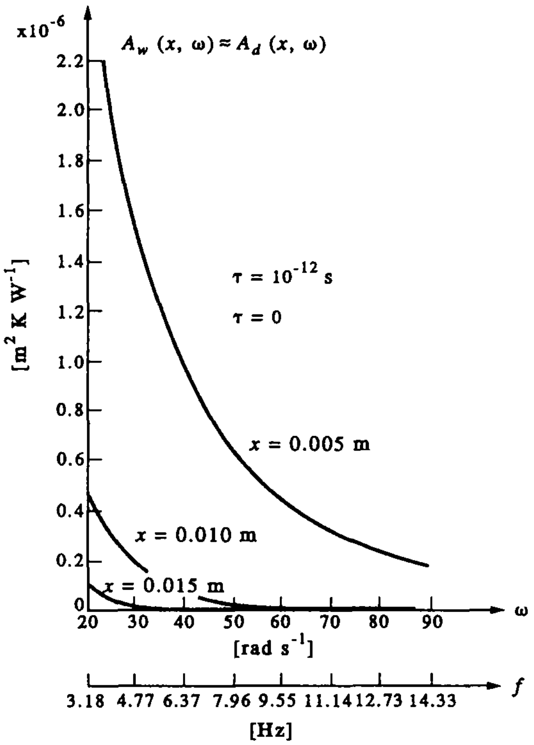

FIG. 1(a). Amplitude characteristics of the wave ($\tau = 10^{-12}$ s) and diffusion ($\tau = 0$) heat flow for $\omega \in \langle 20, 90 \rangle$ rad s^{-1}.

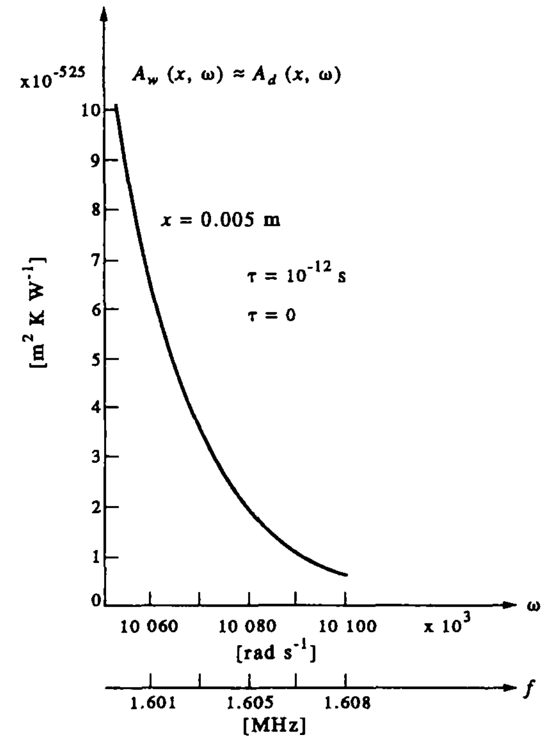

FIG. 1(b). Amplitude characteristics of the wave ($\tau = 10^{-12}$ s) and diffusion ($\tau = 0$) heat flow for $\omega \in \langle 10050, 10100 \rangle$ 10^3 rad s^{-1}.

Furthermore

$$q(x,t) = -k\frac{\partial T(x,t)}{\partial x}. \tag{15b}$$

For the above case we also have to assume boundary conditions (7), (8) and initial conditions (9). The transfer function of the system $K_d(x, s)$ can be determined by using Laplace transformation of (15a) and (15b), or on the basis of relation

$$K_d(x,s) = \lim_{\tau \to 0} K_w(x,s). \tag{16}$$

The transfer function of a semi-infinite solid for diffusion heat flow is as follows:

$$K_d(x,s) = \frac{\sqrt{\alpha}}{k}\frac{1}{\sqrt{s}}\exp\left(-x\sqrt{\left(\frac{s}{\alpha}\right)}\right). \tag{17}$$

The amplitude characteristics take the form

$$A_d(x,\omega) = \frac{\sqrt{\alpha}}{k}\frac{1}{\sqrt{\omega}}\exp\left(-x\sqrt{\left(\frac{\omega}{2\alpha}\right)}\right). \tag{18}$$

Function (18) is monotonically decreasing in relation to ω and reaches the following limit values

$$\lim_{\omega \to 0} A_d(x,\omega) = \infty \tag{19a}$$

$$\lim_{\omega \to \infty} A_d(x,\omega) = 0. \tag{19b}$$

Phase characteristics of the analysed system for diffusion heat flow are as follows:

$$\phi_d(x,\omega) = -\frac{\pi}{4} - x\sqrt{\left(\frac{\omega}{2\alpha}\right)}. \tag{20}$$

NUMERICAL EXAMPLE

We shall, at present, consider a semi-infinite solid. The transfer function of the system was defined by means of formulae (10) and (17). Atypical data were used in our computations [11].

$$k = 145\,[\mathrm{W\,m^{-1}\,K^{-1}}]$$
$$\rho = 2330\,[\mathrm{kg\,m^{-3}}]$$
$$c = 700\,[\mathrm{J\,kg^{-1}\,K^{-1}}]$$
$$\alpha = 8.89 \times 10^{-5}\,[\mathrm{m^2\,s^{-1}}]$$
$$a = 9.429 \times 10^3\,[\mathrm{m\,s^{-1}}]. \tag{21}$$

In the computations the following relaxation time was assumed, $\tau = 10^{-12}$ s [7].

In normal conditions solids are characterized by a very short relaxation time (τ) between 10^{-4} and 10^{-12} s [13, 14]. In the above case the properties of the wave and diffusion media are practically identical in the range of low and medium frequencies (Figs. 1(a) and (b)).

For the parameters assumed above, the limit pulsation is about 10^7 rad s^{-1}. Above this value the differences between the amplitude characteristics values being to increase significantly achieving several hundred percent larger for MHz range (Fig. 2).

The increase of the relaxation time causes a fall in the speed of heat flow which is equivalent to what is appearing in the wave properties of the medium. For example, for $\tau = 10^{-8}$ s the limit pulsation of the amplitude characteristics decreases to about 15×10^3 rad s^{-1}. The above fact results in considerable differences in amplitude characteristics at 4×10^5 rad s^{-1} (Fig. 3). It should be noted here that phase characteristics of the above models are always similar.

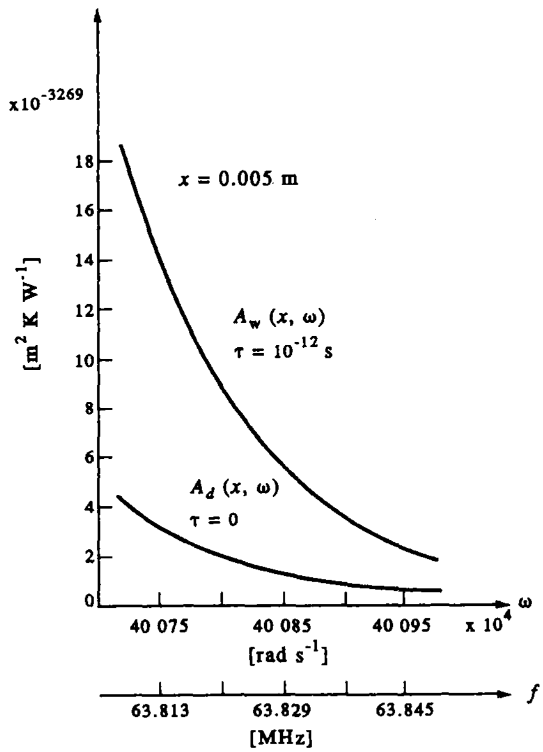

FIG. 2. Amplitude characteristics of the wave ($\tau = 10^{-12}$ s) and diffusion ($\tau = 0$) heat flow for $\omega \in \langle 400\,700,\ 401\,100\rangle$ 10^3 rad s^{-1}.

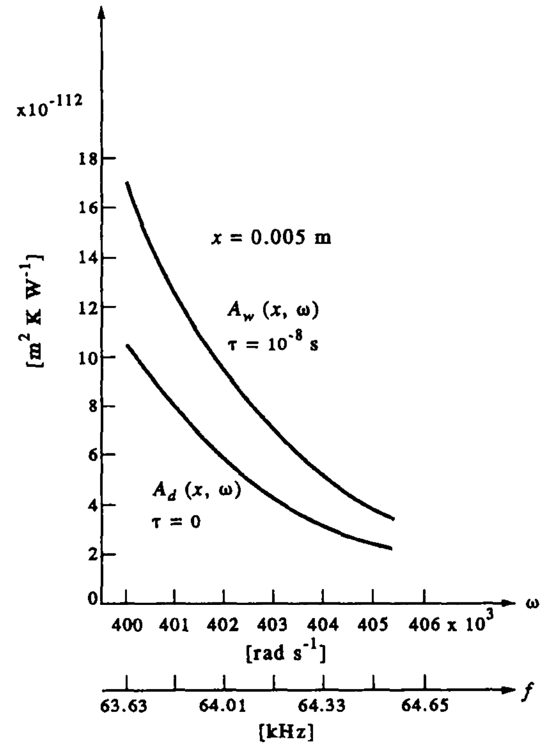

FIG. 3. Amplitude characteristics of the wave ($\tau = 10^{8}$ s) and diffusion ($\tau = 0$) heat flow for $\omega \in \langle 400, 405\rangle$ 10^3 rad s^{-1}.

TRANSIENT TEMPERATURE DISTRIBUTION EXCITED BY AN IDEAL TRIANGULAR IMPULSE

Taking into account theoretical analysis of thermal phenomena in semiconductor devices, the triangular shape of the heat flux striking the semi-infinite solid is of great significance [11]. The shape of the impulse is shown in Fig. 4. Let $1(t)$ be used to define the Heaviside function. The following analytical formula corresponds to the shape form of Fig. 4:

$$q_0(t) = \frac{q_0}{t_0} t 1(t) - \frac{2q_0}{t_0}(t-t_0)1(t-t_0) + \frac{q_0}{t_0}(t-2t_0)1(t-2t_0) \quad (22)$$

with the transform

$$Q_0(s) = \frac{q_0}{t_0}\frac{1}{s^2} - 2\frac{q_0}{t_0}\frac{1}{s^2}e^{-st_0} + \frac{q_0}{t_0}\frac{1}{s^2}e^{-s2t_0}. \quad (23)$$

From equation (3) we obtain

$$\bar{T}(x,s) = F(x,s) - 2F(x,s)e^{-st_0} + F(x,s)e^{-2st_0} \quad (24)$$

where

$$F(x,s) = \frac{q_0}{t_0}\frac{1}{s^2}K_w(x,s) \Leftrightarrow f(x,t). \quad (25)$$

$K_w(x, s)$ represents the transfer function for the wave heat propagation.

Having performed the inverse transformations, the final formula for temperature distribution in the semi-infinite medium is obtained

$$T(x,t) = f(x,t) - 2f(x,t-t_0) + f(x,t-2t_0) \quad (26)$$

where

$$f(x,t) = \frac{q_0}{t_0 k} 1\left(t - \frac{x}{a}\right)\left[\left(\sqrt{(\alpha\tau)} + at\right) \times \int_{x/a}^{t} e^{-\eta/2\tau} I_0\left(\frac{1}{2\tau}\sqrt{\left(\eta^2 - \frac{x^2}{a^2}\right)}\right)d\eta - a\int_{x/a}^{t} \eta\, e^{-\eta/2\tau} I_0\left(\frac{1}{2\tau}\sqrt{\left(\eta^2 - \frac{x^2}{a^2}\right)}\right)d\eta\right]. \quad (27)$$

$I_0(x)$ is a modified Bessel function of the first kind.

For the analysis of the diffusion model formulae, equations (22), (23) and (26) are not changed whereas function $f(x, t) = \psi(x, t)$ takes the form of

$$\psi(x,t) = \frac{q_0\sqrt{\alpha}}{t_0 k} t \int_0^t \frac{1}{\sqrt{(\pi\eta)}}\exp\left(-\frac{x^2}{4\alpha\eta}\right)d\eta - \frac{q_0\sqrt{\alpha}}{t_0 k}\int_0^t \sqrt{\left(\frac{\eta}{\pi}\right)}\exp\left(-\frac{x^2}{4\alpha\eta}\right)d\eta. \quad (28)$$

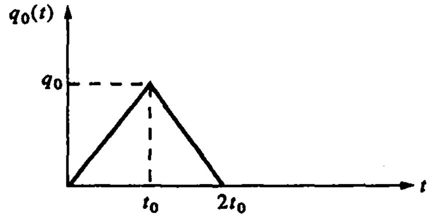

FIG. 4. Ideal triangular impulse of heat flux.

CONCLUSIONS

The transfer functions described by relations (10) and (17) make it possible to determine the transform of temperature at any given point of the half-space considered. The transfer functions mentioned above are irrational and exponent functions of the complex variable s. Transfer functions (10) and (17) make it possible to define the dynamic properties of the medium for the case of wave and diffusion heat flow. In the range of low and medium frequencies, the amplitude characteristics of the half-space analysed are identical for the wave and diffusion heat propagation. However, as soon as the frequency of the heat flux striking the semi-infinite solid increases over a certain limit, the differences between the diffusion and wave heat flow start to show. This fact leads to conclusions concerning the choice of the mathematical model describing the heat propagation in solids.

The wave model of heat propagation is characterized by the so-called dead time for $t \in (0, x/a)$. In this case the thermal wave front will reach a depth x in the time x/a. In the diffusion model the excitations cause immediate perturbations of temperature at point x (equations (27), (28)).

In general, the values of amplitude characteristics are higher for the medium of wave heat propagation. For the quick-changing signals in time the diffusion model gives a considerably lower value (formulae (12b) and (19b), Figs. 2 and 3).

By making use of the results obtained it is possible to determine limit frequencies of both the diffusion and wave models for a single impulse e.g. the triangular shape, depending on its growth time.

Acknowledgement—This work was commissioned by the Polish Ministry of National Education, No DNS-T/06/104/90-2.

REFERENCES

1. C. Cattaneo, A form of heat conduction equation which eliminates the paradox of instantaneous propagation, *C. r. Hebd. Séan. Acad. Sci. Paris* **247**, 431 (1958).
2. P. Vernotte, The true heat equation, *C.r. hebd. Séan. Acad. Sci. Paris* **247**, 2103 (1958).
3. J. I. Frankel, B. Vick and M. N. Özisik, General formulation and analysis of hyperbolic heat conduction in composite media, *Int. J. Heat Mass Transfer* **30**, 1293–1305 (1987).
4. D. E. Glass, M. N. Özisik and B. Vick, Non-Fourier effects on transient temperature resulting from periodic on–off heat flux, *Int. J. Heat Mass Transfers* **30**, 1623–1631 (1987).
5. J. Gembarovič and V. Majernik, Non-Fourier propagation of heat pulses in finite medium, *Int. J. Heat Mass Transfer* **31**, 1073–1080 (1988).
6. W. W. Yuen and S. C. Lee, Non-Fourier heat conduction in a semi-infinite solid subjected to oscillatory surface thermal disturbances, *J. Heat Transfer* **111**, 178–181 (1989).
7. W. Kamiński, Hyperbolic heat conduction equation for materials with a non homogeneous inner structure, *J. Heat Transfer* **112**, 555–560 (1990).
8. M. H. Sadd and Cha Chin Young, Axisymmetric non-Fourier temperatures in cylindrically bounded domains, *Int. J. Non-linear Mechanics* **17**, 129–136 (1982).
9. J. P. Brazel and E. J. Nolan, Non-Fourier effects in the transmission of heat, *Proc. 6th Conf. on Thermal Conductivity*, Dayton, Ohio (1966).
10. A. V. Luikov, *Analytical Heat Diffusion Theory*. Academic Press, New York (1968).
11. V. A. Sankaran, J. L. Hudgins, C. A. Rhodes and W. M. Portnoy, Transient thermal analysis of thyristors using finite element method, *Trans. of IEEE*, PESC'91, Vol. I, 422–428 (1991).
12. V. A. Sankaran, J. L. Hudgins and W. M. Portnoy, Role of the amplifying date structure in the turn-on process of structure thyristors, *IEEE Trans. PEL* **5**, (1990).
13. H. D. Weymann, Finite speed of propagation in heat conduction, diffusion and viscous shear motion, *Am. J. Phys.* **35**, 488–496 (1967).
14. P. M. Kolesnikov, Generalized boundary conditions of the heat and mass transfer, *Int. J. Heat Mass Transfer* **30**, 85–92 (1987).

APPENDIX

Equation (6b) was transformed by means of a Laplace transform, applying the theorems on differentiation and derivative transformation

$$Q(x,s) = -k\frac{\mathrm{d}\bar{T}(x,s)}{\mathrm{d}x} - \tau s Q(x,s) + \tau q(x,0) \qquad \text{(A1)}$$

where $Q(x,s) \Leftrightarrow q(x,t)$. The final term of formula (A1) was eliminated by using formula (9a).

Substituting $x = 0$ after transformations we obtained

$$\left.\frac{\mathrm{d}\bar{T}(x,s)}{\mathrm{d}x}\right|_{x=0} = -\frac{1}{k}(1+s\tau)Q_0(s) \qquad \text{(A2)}$$

where $Q_0(s) = Q(0,s)$ and $q_0(t) = q(0,t)$.

We also transform, in relation to variable t, equation (6a) by applying the theorems mentioned above

$$\alpha\frac{\mathrm{d}^2\bar{T}(x,s)}{\mathrm{d}x^2} = s\bar{T}(x,s) - T(x,0) + \tau s^2\bar{T}(x,s)$$

$$-\tau\left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0} - \tau s T(x,0). \qquad \text{(A3)}$$

By taking into account formulae (9b) and (9c), equation (A3) is considerably simplified

$$\frac{\mathrm{d}^2\bar{T}(x,s)}{\mathrm{d}x^2} - \frac{s(1+s\tau)}{\alpha}\bar{T}(x,s) = 0. \qquad \text{(A4)}$$

A general solution of (A4), with respect to variable x with parameter s, is evident

$$\bar{T}(x,s) = A(s)\exp\left[-\sqrt{\left(\frac{s(1+s\tau)}{\alpha}\right)}x\right]$$

$$+B(s)\exp\left[\sqrt{\left(\frac{s(1+s\tau)}{\alpha}\right)}x\right]. \qquad \text{(A5)}$$

In order to assign an unambiguous value to the root in the exponent, we chose such a branch of the root for which, when $Re\ s \geqslant 0$, we have

$$Re\sqrt{\left(\frac{s(1+s\tau)}{\alpha}\right)} \geqslant 0. \qquad \text{(A6)}$$

Using the theorem on the limit with respect to the second variable for (8) we obtained

$$\lim_{x\to 0}\bar{T}(x,s) = 0. \qquad \text{(A7)}$$

Next equation (A5) was substituted in equation (A7). Taking equation (A6) into consideration we have

$$B(s) = 0. \qquad \text{(A8)}$$

Hence, solution (A5) is reduced to the first term on the right-hand side. Calculating the first derivative of (A5) with respect to the geometrical coordinate ($x = 0$) and considering (A2), after transformations, we have

$$A(s) = \frac{\sqrt{(\alpha\tau)}}{k}\sqrt{\left(\frac{s+(1/\tau)}{s}\right)}Q_0(s). \qquad \text{(A9)}$$

By combining (A9), (A8) and (A5) and applying definition (3), we obtain the transfer function (10) of the wave heat motion in half-space.